TPC 5 dec 2017

Tuesday, December 5, 2017 3:36 PM

TPC 5-dec-2017 Ex. 6

$$\boxed{C(t) \overset{def}{=} \langle \psi_0 | \psi_t \rangle}$$

$\psi_0 = \psi(t=0)$

$\psi_t = \psi(t)$

eigenvalues

$$\boxed{\psi(x,t) = \sum_{m=0}^{\infty} c_m \boxed{\varphi_m(x)} \exp\left(-i \frac{\boxed{E_m}}{\hbar} t\right)}$$

eigenstates $\quad H\varphi_m = E_m \varphi_m$

a) c_m ?

$$\psi(x,t=0) = \sum_{m=0}^{\infty} c_m \varphi_m \cancel{\exp}(0) = \sum c_m \varphi_m$$

we project over $\langle \varphi_n |$

$$\delta_{mn} \begin{cases} 1 & m=n \\ 0 & m \neq n \end{cases}$$

$$\langle \varphi_n | \psi_0 \rangle = \sum_{m=0}^{\infty} c_m \boxed{\langle \varphi_n | \varphi_m \rangle} =$$

$$= c_n$$

$$\Rightarrow \boxed{c_n = \langle \varphi_n | \psi_0 \rangle}$$

b) $$\boxed{F[e^{-i\omega_m t}] = 2\pi\,\delta(\omega - \omega_m)}$$

$$F[f(t)] = \int_{-\infty}^{+\infty} dt\, f(t) \cdot e^{i\omega t}$$

$$\int g(\omega)\,\delta(\omega-\omega_m)\,d\omega = g(\omega_m)$$

$$\psi(x,t) = \sum_m c_m \varphi_m \exp\left(-i\frac{E_m}{\hbar}t\right) \longrightarrow C(t) = \sum_m \underbrace{c_m}_{\langle \varphi_n | \psi_0 \rangle} \langle \psi_0 | \varphi_n \rangle \exp\left(-i\frac{E_m}{\hbar}t\right)$$

$c_n = \langle \varphi_n | \psi_0 \rangle$ $\quad \langle \psi_0 | \psi_t \rangle$ - $\quad c_m^*$

$$\boxed{C(t) = \sum_m |\langle \varphi_n | \psi_0 \rangle|^2 \exp\left(-i\frac{E_m}{\hbar}t\right)}$$

$$\rightarrow \int_{-\infty}^{+\infty} dt \sum_m |\langle \varphi_n | \psi_0 \rangle|^2 \exp\left(-i\frac{E_m}{\hbar}t\right) \cdot e^{i\omega t} =$$

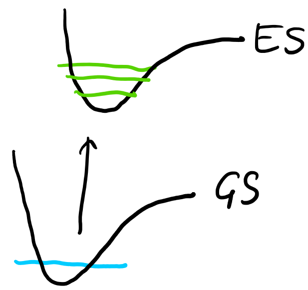
$$= \sum_m |\langle \varphi_n | \psi_0 \rangle|^2 \int_{-\infty}^{+\infty} dt\, \underbrace{\exp\left(-i\boxed{\frac{E_m}{\hbar}}t\right) \exp(i\omega t)} =$$

$$= \sum_m |\langle \varphi_m|\psi_0\rangle|^2 \mathcal{F}\Big[\underbrace{\exp(-i\omega_m t)}_{\to\,\omega_m}\Big] = \Big|\ \omega_m = \frac{E_m}{\hbar}$$

$$= \boxed{\sum_m 2\pi\, |\langle \varphi_m|\psi_0\rangle|^2\, \delta(\omega-\omega_m)}$$

c) $$\boxed{\sigma(\omega) = \frac{2\pi\omega}{3\hbar c}\underbrace{\int_{-\infty}^{+\infty} C(t)\, e^{i\omega t}\, dt}_{\text{see b)}}}$$

$$\boxed{\sigma(\omega) = \frac{4\pi^2\omega}{3\hbar c}\sum_m |\langle \varphi_m|\psi_0\rangle|^2\, \delta(\omega-\omega_m)}$$



d) FERMI GOLDEN RULE

$$\sigma(\omega_I) = \frac{4\pi^2\omega_I}{3\hbar c}\sum_n |\langle\psi_n^E|\hat{\mu}|\psi_0^G\rangle|^2\delta(\omega_I-\omega_n)$$

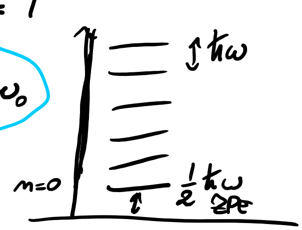
$$\sigma(\omega_I) = \frac{4\pi^2\omega_I}{3\hbar c}\sum_m |\langle\psi_m^E|\psi_0\rangle|^2\delta(\omega_I-\omega_m)$$

$$\psi_0 = \hat{\mu}\psi_0^g$$

e) $$T = \frac{2\pi}{\omega_0}$$

$$H = \frac{1}{2m}p^2 + \frac{1}{2}M\omega_0^2 q^2$$

$$E_m = \left(m+\frac{1}{2}\right)\hbar\omega_0$$



$$C(t) = \sum_m |\langle\varphi_m|\psi_0\rangle|^2 \exp\left(-i\frac{E_m}{\hbar}t\right)$$

$$C(t) = \sum_m |\langle\varphi_m|\psi_0\rangle|^2 \exp\left(-i\left(m+\frac{1}{2}\right)\omega_0 t\right)$$

$$t = k\cdot T = k\,\frac{2\pi}{\omega}$$

$$t = k \cdot T = k \frac{2\pi}{\omega_0}$$

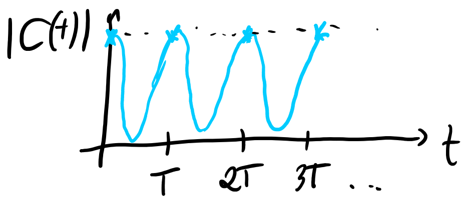
$$C(t) = \sum_n |C_n|^2 \exp\left(-i\left(n+\tfrac{1}{2}\right)\cdot 2\pi \cdot k\right)$$

$$\exp\left(-i \underbrace{(2n+1)\cdot k}\cdot \pi\right) = (-1)^k$$

$$\exp(-i\pi m) = (-1)^m$$

$t=0$

$C(t) = \langle \psi_0 | \psi_0 \rangle = 1$



$$\boxed{T = \frac{2\pi}{\omega_0}}$$

f)

$$\Omega = \frac{2\pi}{T} = \omega_0$$