1) $$|\psi^{(0)}(t)\rangle = U_0(t-t_0)\,|\psi(t_0)\rangle$$

$$|\psi(t_0)\rangle \to |\phi_0\rangle \otimes |G\rangle$$

$$U_0(t-t_0) \to |G\rangle U_0^G(t-t_0)\langle G| + |X\rangle U_0^X(t-t_0)\langle X|$$

$$U_0(t-t_0)|\psi(t_0)\rangle = U_0^G(t-t_0)|\phi_0\rangle \otimes \underbrace{\langle G|G\rangle}_{=1}|G\rangle +$$

$$+\, U_0^X(t-t_0)|\phi_0\rangle \otimes \underbrace{\langle X|G\rangle}_{=0}|X\rangle$$

for orthonormalization of the adiabatic states

so finally

$$\boxed{|\psi^{(0)}\rangle = U_0^G(t-t_0)|\phi_0\rangle \otimes |G\rangle}$$

$$|\psi^{(1)}(t)\rangle = \left(-\frac{i}{\hbar}\right)\int_{-\infty}^{t} d\tau\, \hat{U}_0(t-\tau)\left(\hat{\mu}E(t)\right)\hat{U}_0(\tau-t_0)\,|\psi(t_0)\rangle$$

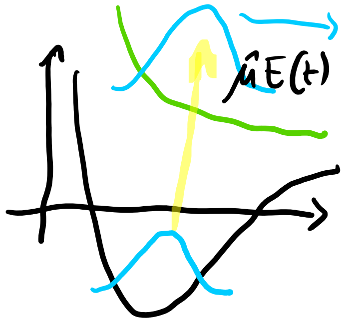
$$\hat{\mu} = \mu_e\left(|G\rangle\langle X| + |X\rangle\langle G|\right)$$

$$= \left(-\frac{i}{\hbar}\right)\int_{-\infty}^{t} d\tau\, \hat{U}_0(t-\tau)\left(\hat{\mu}E(t)\right)\left(U_0^G(t-t_0)|\phi_0\rangle \otimes |G\rangle\right) =$$

$$= \left(-\frac{i}{\hbar}\right)\int_{-\infty}^{t} d\tau\, \hat{U}_0(t-\tau)\,E(t)\left(\hat{U}_0^G(t-t_0)|\phi_0\rangle \otimes |X\rangle\right)\mu_e =$$

$$= \left(-\frac{i}{\hbar}\right)\int_{-\infty}^{t} d\tau\, \mu_e E(t)\left(\hat{U}_0^X(t-\tau)\,\hat{U}_0^G(t-t_0)|\phi_0\rangle \otimes |X\rangle\right) =$$

$$= \left(-\frac{i}{\hbar}\right)\left(\int_{-\infty}^{t} d\tau \underbrace{\hat{U}_0^X(t-\tau)}_{3)} \underbrace{\mu_e E(t)}_{2)} \underbrace{\hat{U}_0^G(\tau-t_0)|\psi_0\rangle}_{1)}\right) \otimes |X\rangle$$



1) propagation on the ground state
2) interaction with the field
   $\rightarrow$ excitation from G to X
3) propagation on the excited state

1b)

$$\hat{U}_0^G(\tau - t_0)|\psi_0\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}^G(\tau-t_0)\right)|\psi_0\rangle$$

$$\hat{H}^G|\psi_0\rangle = E_0|\psi_0\rangle$$

NOTE: if $|a\rangle$ is eigenvector of A with a eigenvalue, it is also eigenvector of $f(A)$ with $f(a)$ eigenvalue

$$A|a\rangle = a|a\rangle \rightarrow f(A)|a\rangle = f(a)|a\rangle$$

$$\hat{U}_0^G(\tau - t_0)|\psi_0\rangle = \exp\left(-\frac{i}{\hbar}E_0(\tau-t_0)\right)|\psi_0\rangle$$

$$\hat{U}_0^G(t - t_0)|\psi_0\rangle = \exp\left(-\frac{i}{\hbar}E_0(t-t_0)\right)|\psi_0\rangle.$$

2)

$$P(t) = \underset{\sim E^0}{P^{(0)}(t)} + \underset{\sim E^1}{P^{(1)}(t)} + \underset{\sim E^2}{P^{(2)}(t)} + \underset{\sim E^3}{P^{(3)}(t)} \dots$$

$$|\psi\rangle = \underset{\sim E^0}{|\psi^{(0)}\rangle} + \underset{\sim E^1}{|\psi^{(1)}\rangle} + \underset{\sim E^2}{|\psi^{(2)}\rangle} \dots$$

$$P(t) = \langle\psi(t)|\hat{\mu}|\psi(t)\rangle = \underset{\sim E^0(t)}{\langle\Phi^{(0)}|\hat{\mu}|\Phi^{(0)}\rangle} + \underset{\sim E^1(t)}{\langle\Phi^{(1)}|\hat{\mu}|\Phi^{(0)}\rangle} + \dots$$

$$+ \langle \Psi^{(0)} | \hat{\mu} | \Psi^{(1)} \rangle + \langle \Psi^{(1)} | \hat{\mu} | \Psi^{(1)} \rangle + \langle \Psi^{(2)} | \hat{\mu} | \Psi^{(0)} \rangle + \dots$$

$\sim E^1(t)$ $\quad \sim E^0(t)$ $\quad \sim E^1(t)$ $\quad \sim E^2(t)$ $\quad \sim E^2(t)$

$$p^{(1)}(t) = \langle \Psi^{(1)} | \hat{\mu} | \Psi^{(0)} \rangle + \underbrace{\langle \Psi^{(0)} | \hat{\mu} | \Psi^{(1)} \rangle}_{c.c.}$$

$$a + a^* = 2 \operatorname{Re} a$$

$$\boxed{p^{(1)}(t) = 2 \operatorname{Re} \langle \Psi^{(1)} | \hat{\mu} | \Psi^{(0)} \rangle}$$

3) $\langle \Psi^{(0)} | \hat{\mu} | \Psi^{(1)} \rangle$ eq.s (7) and (8)

$$|\Psi^{(1)}\rangle = |\psi^{(1)}(t)\rangle \otimes |X\rangle$$

$$|\Psi^{(0)}\rangle = |\psi^{(0)}(t)\rangle \otimes |G\rangle$$

$$\hat{\mu} = \mu_e \left(|X\rangle\langle G| + |G\rangle\langle X| \right)$$

$$\langle \Psi^{(0)} | \hat{\mu} | \Psi^{(1)} \rangle = \langle \psi^{(0)}(t) | \mu_e | \psi^{(1)}(t) \rangle$$

$$|\psi^{(0)}\rangle = \exp\left(-\frac{i}{\hbar} E_0 (t - t_0)\right) |\psi_0\rangle$$

$$|\psi^{(1)}\rangle = \left(-\frac{i}{\hbar}\right) \int_{-\infty}^{t} d\tau \, \hat{U}_0^X(t-\tau) \left(\mu_e E(\tau)\right) \exp\left(-\frac{i}{\hbar} E_0 (\tau - t_0)\right) |\psi_0\rangle$$

$$\langle \psi^{(0)}(t) | \mu_e | \psi^{(1)}(t) \rangle = \left(-\frac{i}{\hbar}\right) \int_{-\infty}^{t} d\tau \, \langle \psi_0 | \hat{U}_0^X(t-\tau) \mu_e^2 | \psi_0 \rangle \cdot$$

$$\cdot \exp\left(-\frac{i}{\hbar} E_0 \left[(\tau - t_0) - (t - t_0) \right] \right) \cdot E(\tau) =$$

$$= \left(-\frac{i}{\hbar}\right) \int_{-\infty}^{t} d\tau \, \underbrace{\langle \psi_0 | \hat{U}_0^{\dagger}(t-\tau) \mu_e^2 | \psi_0 \rangle}_{\downarrow} \exp\left(-\frac{i}{\hbar} E_0 (\tau - t)\right) E(\tau)$$

$$\mu_e^2 \langle \psi_0 | \hat{U}_0^{\dagger}(t-\tau) | \psi_0 \rangle = C(t-\tau)$$

$$\boxed{\tau \longrightarrow \tilde{t} = t - \tau}$$

$$d\tau = -d\tilde{t}$$

$$-\infty \rightarrow +\infty$$

$$t \rightarrow 0$$

change of integration variables!

$$= \boxed{\left(-\frac{i}{\hbar}\right) \int_{0}^{+\infty} d\tilde{t} \, C(\tilde{t}) \exp\left(+\frac{i}{\hbar} E_0 \tilde{t}\right) E(t-\tilde{t})}$$